Pure-Past Linear Temporal and Dynamic Logic on Finite Traces

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Abstract

We review PLTL_f and PLDL_f, the pure-past versions of the well-known logics on finite traces LTL_f and LDL_f, respectively. PLTL_f and PLDL_f are logics about the past, and so scan the trace backwards from the end towards the beginning. Because of this, we can exploit a foundational result on reverse languages to get an exponential improvement, over $\text{LTL}_f/\text{LDL}_f$, for computing the corresponding DFA. This exponential improvement is reflected in several forms of sequential decision making involving temporal specifications, such as planning and decision problems in non-deterministic and non-Markovian domains. Interestingly, PLTL_f (resp., PLDL_f) has the same expressive power as LTL_f (resp., LDL_f), but transforming a PLTL_f (resp., PLDL_f) formula into its equivalent LTL_f (resp., LDL_f) is quite expensive. Hence, to take advantage of the exponential improvement, properties of interest must be directly expressed in $\text{PLTL}_f/\text{PLDL}_f$.

LTL_f and LDL_f

LTL_f is a variant of Linear-time Temporal Logic (LTL) interpreted on *finite*, instead of infinite, traces [1]. Given a set $\mathcal{P}$ of atomic propositions, LTL_f formulas φ are defined by:

$$\varphi ::= a \mid \neg\varphi \mid \varphi \wedge \varphi \mid \bigcirc\varphi \mid \varphi\,\mathcal{U}\,\varphi$$

where $a \in \mathcal{P}$, $\bigcirc$ is the **next** operator, and $\mathcal{U}$ is the **until** operator. Derived future temporal operators are: **eventually** $\Diamond\varphi \doteq \top\,\mathcal{U}\,\varphi$; **always** $\Box\varphi \doteq \neg\Diamond\neg\varphi$; and **weak next** $\bullet\varphi \doteq \neg\bigcirc\neg\varphi$.

LDL_f is a proper extension of LTL_f that captures regular expressions on finite traces.

$$\varphi ::= tt \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\varrho\rangle\varphi$$
$$\varrho ::= \phi \mid \varphi? \mid \varrho + \varrho \mid \varrho;\varrho \mid \varrho^*$$

where tt denotes the LDL_f true formula, ϕ propositional formulas over $\mathcal{P}$, ϱ path expressions, and $\varphi?$ the test construct. Derived operators are: $[\varrho]\varphi \doteq \neg\langle\varrho\rangle\neg\varphi$, $ff \doteq \neg tt$, and $end = [true]ff$. Intuitively, $\langle\varrho\rangle\varphi$ states that there exists an execution satisfying the RE ϱ such that its last step satisfies φ; whereas $[\rho]\varphi$ states that, from the current step, all executions satisfying the RE ϱ are such that their last step satisfies φ.

PLTL_f and PLDL_f

PLTL_f is the *pure-past* version of LTL_f. PLTL_f formulas are satisfied if they hold in the last instant of the trace. Given the set $\mathcal{P}$, PLTL_f formulas are defined as:

$$\varphi ::= a \mid \neg\varphi \mid \varphi \wedge \varphi \mid \ominus\varphi \mid \varphi\,\mathcal{S}\,\varphi$$

where $a \in \mathcal{P}$, $\ominus$ is the **previous** operator, and $\mathcal{S}$ is the **since** operator. Derived past temporal operators are: **once** $\diamondminus\varphi \doteq \top\,\mathcal{S}\,\varphi$; **historically** $\boxminus\varphi \doteq \neg\diamondminus\neg\varphi$.

PLDL_f is a proper extension of PLTL_f, hence the *pure-past* version of LDL_f.

$$\varphi ::= tt \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\!\langle\varrho\rangle\!\rangle\varphi$$
$$\varrho ::= \phi \mid \varphi? \mid \varrho + \varrho \mid \varrho;\varrho \mid \varrho^*$$

In PLDL_f, the LDL_f diamond operator is replaced by a backward diamond operator. Intuitively, $\langle\!\langle\varrho\rangle\!\rangle\varphi$ states that there exists a point in the past, reachable (going backwards) through the RE ϱ from the current instant, where φ holds. Derived operators are: $[\![\varrho]\!]\varphi \doteq \neg\langle\!\langle\varrho\rangle\!\rangle\neg\varphi$, $ff \doteq \neg tt$, and $start = [\![true]\!]ff$. Intuitively, $[\![\varrho]\!]\varphi$ states that, from the current step, all executions satisfying the RE ϱ (going backwards) are such that their last step in the past satisfies φ.

Examples

"every time you took the bus, you bought a new ticket beforehand"

- PLTL_f: $\boxminus(takeB \supset \ominus(\neg takeB\,\mathcal{S}\,buyT))$
- LTL_f: $(buyT\,\mathcal{R}\,takeB) \wedge \Box(takeB \supset (buyT \vee \bigcirc(buyT\,\mathcal{R}\,\neg takeB)))$

"every time, if the cargo-ship departed (cs), then beforehand there was an alternation of grab and unload (unl) of containers"

- PLDL_f: $[\![true^*]\!](\langle\!\langle cs\rangle\!\rangle tt \supset \langle\!\langle(unl;grab)^*;(unl;grab)\rangle\!\rangle start)$
- LDL_f: $\langle(\neg cs + (grab \wedge \neg cs);(unl;(grab \wedge \neg cs))^*;(cs \wedge unl));\neg cs^*\rangle end$

From φ to Automata

- For $\text{LTL}_f/\text{LDL}_f$ formulas the translation is worst-case 2EXPTIME:

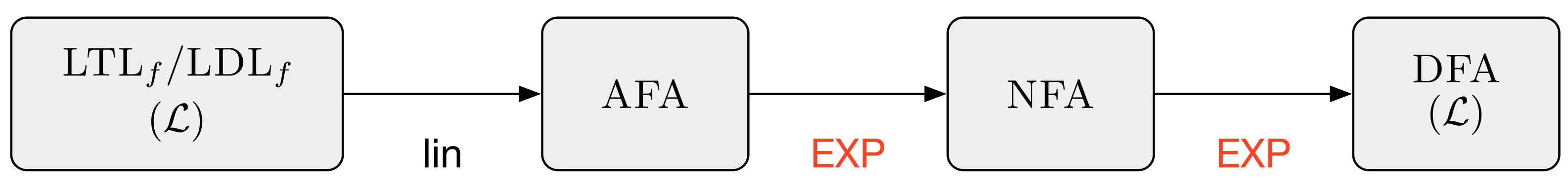


Figure 1: Translation algorithm from $\text{LTL}_f/\text{LDL}_f$ formulas to DFA

- For $\text{PLTL}_f/\text{PLDL}_f$ formulas the translation is worst-case EXPTIME:

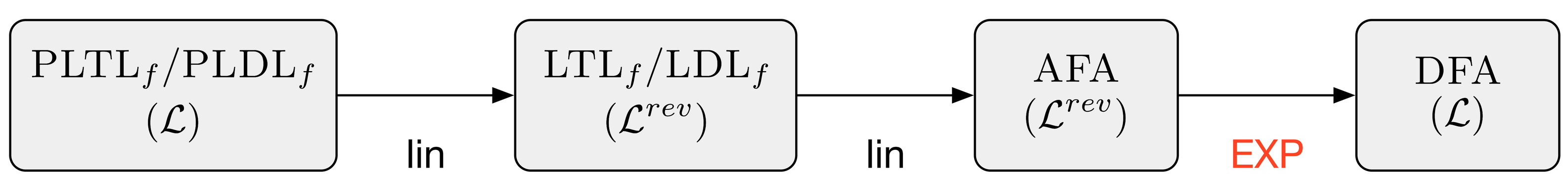
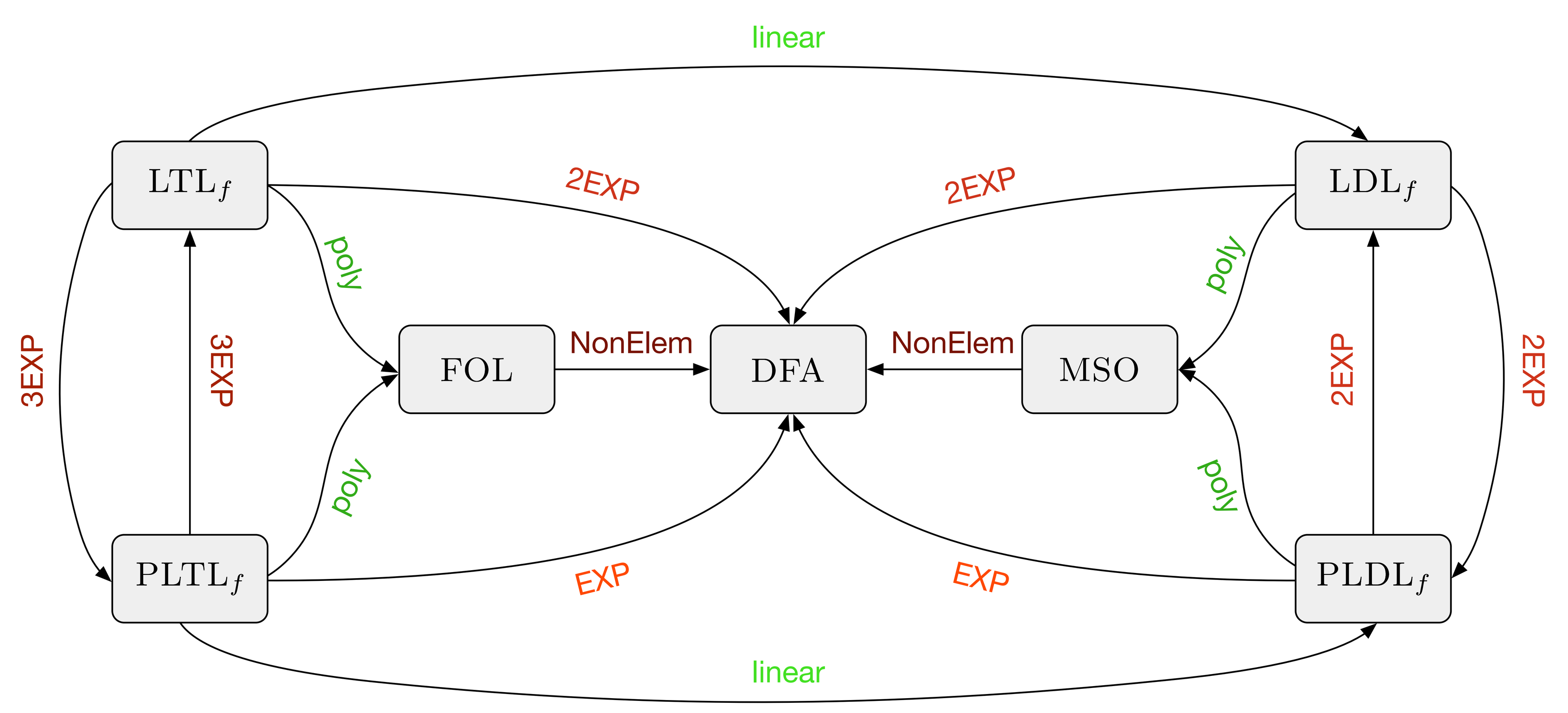


Figure 2: Translation algorithm from $\text{PLTL}_f/\text{PLDL}_f$ formulas to DFA

This is due to a fundamental property of *alternating finite-state automata* (AFA) [2], for which one can obtain directly the DFA of the *reverse* language, namely moving from a past view of the trace to a future one.

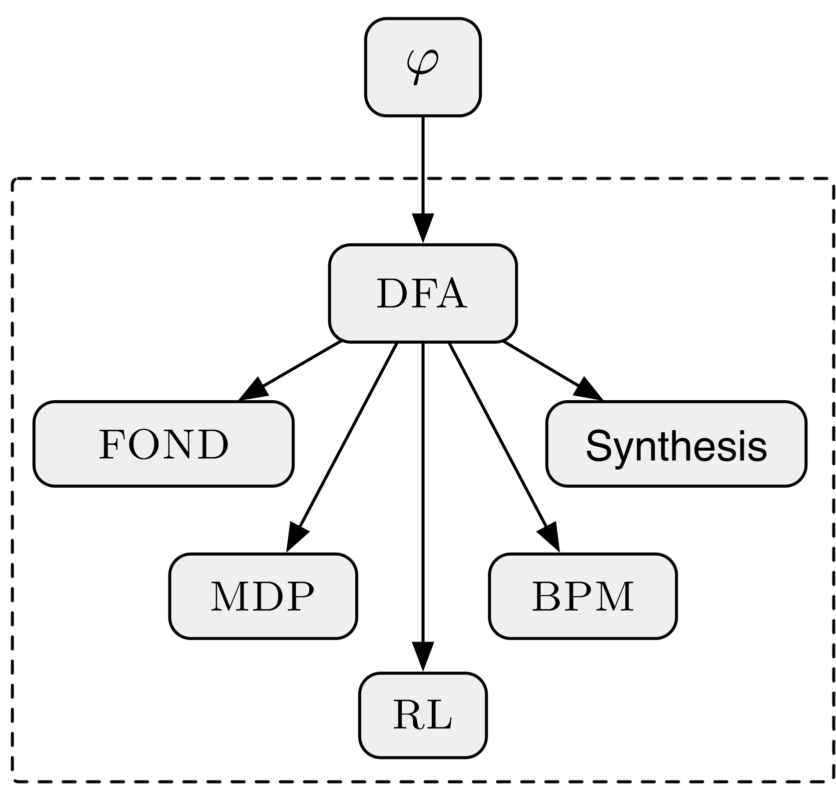
Transformations

Here, we summarize all the transformations and the relationship between $\text{LTL}_f/\text{LDL}_f$ and $\text{PLTL}_f/\text{PLDL}_f$ that we reviewed in the paper.



Implications

The exponential gain in transforming $\text{PLTL}_f/\text{PLDL}_f$ formulas into DFAs, with respect to $\text{LTL}_f/\text{LDL}_f$, is reflected in an exponential gain in solving several forms of sequential decision making problems involving temporal specifications.



Takeaways

1. If you can *naturally* express the specification in $\text{PLTL}_f/\text{PLDL}_f$, then do it to get the computational advantage
2. Converting $\text{LTL}_f/\text{LDL}_f$ to $\text{PLTL}_f/\text{PLDL}_f$ to get the exponential advantage is **not** computationally sensible
3. Complexities are just worst-case, in most AI applications the size of the resulting DFA is actually manageable

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References

[1] G. De Giacomo and M. Vardi.
Linear temporal logic and linear dynamic logic on finite traces.
In *IJCAI*, 2013.

[2] A. Chandra, D. Kozen, and L. Stockmeyer.
Alternation.
J. of the ACM, 28(1), 1981.

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